

Engineering polar vortices via strain soliton interactions in marginally twisted multilayer graphene

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ABSTRACT

Strain solitons have been widely observed in van der Waals materials and their heterostructures. They can manifest as one-dimensional (1D) wires and quasi two-dimensional (2D) networks. However, their coexistence within the same region has rarely been observed and their interplay remains unexplored. Here, employing lateral piezoresponse force microscopy, we show 1D linear solitons and 2D moiré solitons appear simultaneously and interact in twisted multilayer graphene. In twisted monolayer-bilayer graphene, when a linear soliton intersects with moiré solitons, the polarization reverses across the intersections, splitting a polar vortex into two vortices. Interestingly, when a linear soliton is parallel to a moiré soliton, they can annihilate each other in certain cases, resulting in the unification of polar vortices. In twisted monolayer-trilayer graphene,

linear solitons from different interfaces can be resolved. Our results provide new insights for the interplay between different solitons, and pave a new way for engineering moiré physics.

KEYWORDS: strain solitons, moiré superlattice, lateral piezoresponse force microscopy, polar vortex, soliton annihilation

Dislocations in van der Waals (vdW) materials and their heterostructures play a crucial role in defining their electronic, optical and mechanical properties¹⁻¹⁴. Among these, interlayer dislocations, arising from lattice mismatch, strain, or twist, can induce localized deformations that propagate as strain solitons, or soliton-like stacking domain walls, within the material. In bilayer graphene, these strain solitons separating different stacking (AB and BA) domains, termed linear solitons, have been shown to exhibit the quantum valley Hall effects and linear magnetoresistance^{6,7}. High-resolution transmission electron microscopy and scanning near-field optical microscopy have revealed linear solitons of different types, i.e. shear, tensile and mixed^{3,8}, depending on the orientation of the interlayer displacement (Burgers) vector relative to the soliton boundary. These linear solitons can be manipulated by heating up to higher temperatures, applying an electrostatic gate or external mechanical stress etc^{3,8,11,12,15,16}.

Beyond natural vdW materials, interlayer dislocations can also be engineered through controlled twisting, which generates nonuniform strains within the layers. This approach has led to the discovery of various exotic, emergent correlated and topological phenomena in twisted vdW materials¹⁷⁻²³, which can be finely tuned by adjusting the twist angle. In particular, in marginally twisted bilayer graphene (tBLG), lattice reconstruction can give rise to alternating Bernal-stacked AB, BA domains and topological point defects (AA stacking domains)^{24,25}. In this regime, the dislocations are confined to the domain wall boundaries, creating moiré solitons²⁶. These moiré solitons are primarily shear-type^{3,8,16,26}, where the strain-induced electric polarizations are aligned along the moiré solitons. When electron energy gap forms in AB and BA regions by an out-of-plane electric field, the moiré solitons can host topologically protected valley helical edge states, minibands inside the gap and Aharonov-Bohm oscillations²⁷⁻²⁹. Additionally, recent studies on

twisted MoS₂ and WSe₂ bilayers reveal that these dislocations can host complex polar vortex patterns^{30,31}.

While the coexistence of linear and moiré solitons in marginally twisted multilayer graphene may seem self-evident given their shared origin as interlayer dislocations, only a couple of studies have documented their coexistence within the same regions³²⁻³⁴. Moreover, the intricate interplay between these solitons, as well as its effects on their piezoelectric responses and topological polar textures, has not yet been investigated. In this work, we examine a series of marginally tMBG and tMTG samples (tMBG1 to tMBG5 and tMTG1 to tMTG5), most of which reveals the simultaneous presence of linear and moiré solitons. By investigating their electromechanical responses and stacking configurations, we uncover a diverse range of effects arising from the interplay between these solitons.

We first study Bernal-stacked bilayer graphene stacked atop monolayer graphene with twist-angles smaller than 0.1 degree (moiré period ~140 nm) (see Methods and Fig. S1 for details of device fabrication). Figure 1a&b depict the schematic measurement setup and stacking configurations of tMBG moiré superlattice, where AB-B, AB-A and AB-C (labeled from top to bottom layers and hyphens for the twisted interfaces) represent three different stacking domains. The moiré solitons are marked by the brown regions that bridge adjacent AB-B domains. LPFM is utilized to investigate the in-plane electromechanical coupling of tMBG through inverse piezoelectric effect, which captures projected lateral deformation or electric polarization perpendicular to the cantilever^{35,36}.

Figure 1c plots the LPFM phase image (decoupled from the background signals, details of the decoupling procedures can be found in the Supplementary Note 1 and Fig. S2) of a tMBG sample (tMBG1), representing a typical tMBG moiré superlattice. Similar to tBLG³⁵ and tDBG³⁶, strong contrasts are observed near moiré solitons as a result of the localization of strain. The stacking domains labeled by the green and blue dots are inferred from the curvature of the domains in according with the stacking energy difference between AB-A and AB-C stackings (see Fig. S3 for stacking energy calculations^{32,37-39}). Obviously, along the same directions, moiré solitons exhibit

similar phases and magnitudes. Raw and decoupled images of the amplitudes and phases before and after rotating the sample by 90° can be found in Fig. S4. The complete lateral deformation or in-plane polarization image of the tMBG can be reconstructed by combining measurements along two independent directions, as shown in Fig. S4e. The interconnected in-plane polarization suggests the formation of chiral polar vortex network in tMBG.

Apart from typical tMBG samples like tMBG1, some tMBG samples show distinct LPFM signal variations. In Fig. 1d, for instance, we can observe additional pronounced contrast along the red dashed line that intersects the triangular moiré superlattice network in tMBG2. Notably, across the intersections, the LPFM phase always changes by $\sim 180^\circ$ along both the red dashed line and moiré solitons. Figure 1e plots linecuts of the LPFM phase and amplitude along the white arrowed line in Fig. 1d. While the phase changes by 180° across the intersection, the amplitude remains approximately constant except near the intersections, indicating the switching of LPFM signal and therefore in-plane electric polarization across the intersections. Images of the same area, with the sample rotated, are provided in Fig. S5. The angular dependence suggests the lateral deformation of the linear soliton is along the soliton direction (Supplementary Note 2). We then plot the schematic reconstructed deformation map in Fig. 1f. We also see each moiré stacking domain intersected by the red dashed linear is split into two subdomains of opposite curvatures (convex and concave, as inferred from the nearby triangular domains), indicating different stackings configurations. Triangular domains with the same stacking type (Bernal or rhombohedral) on either side of the red dashed line are oriented in opposite directions, in contrast to tMBG1. The change of stacking configurations can also be visualized in the cAFM map (Fig. S6). Thus, the red dashed line represents a 1D linear soliton, and an intersected polar vortex would split into two polar vortices of opposite chiralities. The schematics of clockwise and counterclockwise vortices are illustrated in Fig. 1g.

The in-plane electric polarizations at DWs of twisted graphene systems are found to arise from the piezoelectric or flexoelectric effect. If the linear soliton is located at the bilayer interface, the displacement vector and thus the strain field would remain similar across the intersections.

Therefore, the switching of electric polarization of linear and moiré solitons is driven by changes in the atomic stacking configurations. To understand this, we examine the stacking configurations near the center of Fig. 1d. If the top bilayer is AB and BA stacked on the left and right sides of the linear soliton, the atomic arrangements of domains near the intersection can be sketched as shown in the insets of Fig. 1f. When rotated around the out-of-plane direction by 180° , the atomic arrangements are restored. The above symmetry analysis suggests that the in-plane electric polarizations would be reversed across the intersection for both the linear and moiré solitons. Similar conclusions can be drawn for linear solitons with the opposite Burgers vector.

Figure 2a&d present additional LPFM images of tMBG samples with linear solitons. Across the intersections, the in-plane electric polarization of linear and moiré solitons also reverses. The linear solitons are found to locate either near the middle of two parallel moiré solitons (tMBG3, Fig. 2a) or in close proximity to a single moiré soliton (tMBG4, Fig. 2d), suggesting effective repulsive or attractive interactions, respectively. These two cases are distinct by the Burgers vector orientation of the linear solitons, which results in different stacking configurations shown in the insets of Fig. 2b&e. Using a simplified model, we plot the calculated total stacking energies (Fig. 2b&e) as a function of the relative position of the linear soliton in both cases (see Methods for details). The energies reach their minimum (steady states) when the linear solitons are located at the midpoint between moiré solitons and directly at the moiré solitons, consistent with the observed structures in Fig. 2a&d, respectively. We also perform molecular dynamics (MD) simulations on the system. In the MD simulation, we start with rigid atomic stackings, and place a linear soliton between two parallel moiré solitons, and allow them relax together (Fig. S7). The obtained domain structures (Fig. 2c&f) are also consistent with the corresponding LPFM images. We note linear solitons usually appear at the vicinity of structural defects or sample boundaries, which may act as anchor points for them. In practice, the strain solitons and their interactions may be complicated by these boundary conditions.

In the above analysis, we assume that linear solitons are embedded within the bilayer graphene, preventing them from merging with moiré solitons at the twisted interface, even when in close

proximity. Interestingly, in a couple of tMBG samples, we observe linear solitons merging with moiré solitons, displaying nearly vanishing LPFM signal. Figure 2g shows the phase image of tMBG5, in which the linear soliton is marked by the red dashed line. No LPFM signal can be measured on the linear solitons, irrespective of their orientation relative to the cantilever (see Fig. S8 for the angular dependence). In Fig. 2h, we show the cAFM current image of the same area with a slight distortion of the moiré pattern. The linecuts in Fig. 2i confirm the unification of adjacent triangular domains across the linear soliton, resulting in diamond-shaped domains. The annihilation of linear and moiré solitons also leads to the unification of polar vortices.

We propose the linear soliton in sample tMBG5 is a shear soliton residing at the twisted interface. The displacement vector is along the shear soliton, i.e. an armchair direction of graphene. Since the displacement vector of a moiré soliton is also nearly along an armchair direction of the constituent graphene, it is possible the two displacements would be the opposite and add up to zero, eliminating the strain and the AB-A/AB-C domain walls (Fig. S9). To obtain the resulting moiré pattern, nonuniform displacement vectors are demanded to translate AB-A to AB-C and AB-C to AB-A stackings. This may happen during the stacking process or be triggered by an AFM tip after sample preparation, which has been reported in twisted multilayer graphene before³³. Consequently, the displacement vectors along the same directions are opposite on the two sides of the shear solitons. From the constitutive equation $P_i = e_{ijk}\epsilon_{jk} + \mu_{ijkl}\frac{\partial\epsilon_{jk}}{\partial x_l}$, where e_{ijk} is the piezoelectric coefficient, μ_{ijkl} is the flexoelectric coefficient, $\epsilon_{jk} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}\right)$ is the strain field, $\vec{u}$ is the displacement field, the strain and in-plane electric polarization of moiré solitons would be reversed across the shear soliton.

For tMBG, if a linear soliton is neither in proximity to nor merging with moiré solitons, similar observations would be made for linear solitons located within the bilayer and at the twisted interface in LPFM/cAFM. Thus, more sophisticated scanning probe technique would be required to determine the depth of the linear soliton. The scenario is different for marginally tMTG. As illustrated in Fig. 3a, if a linear soliton is located between the upper two layers of the trilayer graphene (the first interface from the top), and the trilayer graphene is ABA stacked, the stacking

orders would change from ABA-B to BAC-A and from ABA-C to BAC-B. This gives rise to four distinct stacking configurations, aligning with the four contrasting currents observed in Fig. 3b for tMTG1. From the relative stacking energies of different domains³⁹ (Fig. S3), we determine their curvatures maintain (concave or convex), which also agrees with Fig. 3b. If the linear soliton is located between the lower two layers of the trilayer graphene (the second interface, as illustrated in Fig. 3c), the stacking orders would change from ABA-B to ABC-A and from ABA-C to ABC-B. The current map would also display four distinct currents, but the relative curvatures differ, as demonstrated in Fig. 3d for tMTG2.

On the other hand, if the linear soliton is positioned at the twisted interface (the third interface, as illustrated in Fig. 3e), then there would only be two stacking domains after relaxation, i.e. concave ABA-B domains and convex ABA-C domains on either side of the linear soliton. Indeed, in the current map of tMTG3 (Fig. 3f), only two contrasting currents are observed, and the linear soliton merges with the moiré solitons. The slight current enhancement near the linear soliton might arise from the imperfect annihilation of the linear and moiré solitons. Stacking energy analysis of linear solitons in tMTG is provided in Fig. S10. In short, the combination of curvatures and current changes of the stacking domains enables us to determine the depth of linear solitons in tMTG.

We also note the piezoelectric response of linear and moiré solitons in tMTG is more complex than that in tMBG, due to the reduced symmetry in tMTG. In Fig. 4a, we show the current map of tMTG4, in which two linear solitons embedded at different interfaces of the trilayer graphene can be identified. The sample is divided into three different regions by them, marked by red dashed lines. Considering the higher averaged current in Region II, we sketch the stacking configurations of each region in Fig. 4b. Figure 4c shows the corresponding decoupled LPFM phase image (see raw and decoupled amplitude images in Fig. S11). Overall, when linear and moiré solitons intersect in tMTG, their piezoelectric responses appear to depend on the depth of the linear solitons. Specifically, the in-plane electric polarizations of moiré solitons reverse across the linear soliton at the first interface, which separates Regions II and III. In contrast, for linear solitons from the

second interface intersect the moiré solitons, separating Regions I and II, the electric polarization of both linear and moiré solitons retains the same polarities. Along with variations in polarity, the LPFM amplitude in tMTG may also exhibit changes at intersections, leading to a more complex evolution of the polar texture. Additionally, for linear solitons located at the twisted interface, the overall piezoelectric responses of the linear and moiré solitons (Fig. S12) resemble those observed in tMBG.

Finally, we demonstrate the linear solitons in tMBG and tMTG are more mobile than moiré solitons. Figure 4d presents selected frames of LPFM phase captured over multiple scans of tMTG5. The positions of the linear soliton are outlined by the dashed lines in Fig. 4e, and the arrows mark the subsequent motions of the linear soliton. While moiré solitons also appear to move, the linear soliton exhibits more pronounced movement within the moiré superlattice before being expelled to the sample boundary. Figure S13 also shows motions of a linear soliton in tMBG, which tends to merge with the sample boundary over multiple scans. These observations underscore the potential for engineering a moiré superlattice by manipulating the movement of linear solitons, either using an AFM tip or through other methods.

In summary, the incorporation of a 1D soliton in a 2D moiré superlattice can generate a rich landscape of stacking configurations, depending on the Burgers vector, the relative depth of the linear soliton and other factors like defects and boundaries. The interplay between linear and moiré solitons in tMBG can lead to annihilation of solitons, switching of electric polarizations and rearrangement of polar vortices. In tMTG, linear solitons from different interfaces can be resolved by combining domain curvatures and current analysis. We anticipate these results can be extended to other moiré systems, such as those exhibiting moiré ferroelectricity⁴⁰⁻⁴². Further investigation of electronic and topological properties near the 1D solitons can open up new opportunities for functional devices by exploiting the exceptional tunability of linear solitons.

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Sample fabrication. All devices were fabricated using a modified ‘tear-and-stack’ method and the preparation processes are schematically shown in Fig. S1a. A polycarbonate (PC) film on top of a polydimethylsiloxane (PDMS) stamp was used to pick up h-BN, then the monolayer graphene part of a monolayer-bilayer (monolayer-trilayer) graphene junction, followed by the bilayer (trilayer) graphene part with a desired twist angle. Another few-layer graphene is picked up as an electrical contact for twisted graphene sample. To access the surface of tMBG and tMTG, the PC film was peeled off from stamp, and then flipped over and placed on a Si/SiO₂ substrate. Direct electrical contact was made to the twisted graphene sample via silver paste.

Atomic force microscopy measurements. LPFM and cAFM modes were adopted in this work, both conducted on an Oxford Instruments Asylum Research MFP-3D Origin AFM. In LPFM, Ti/Ir coated silicon probes (ASYELEC.01-R2) with a spring constant of ~ 2.8 N/m (free resonance frequency ~ 75 kHz) were used to perform single frequency LPFM at a lateral resonance frequency of ~ 720 kHz. In cAFM, Pt coated silicon probes (HQ:NSC18/Pt) with a spring constant of ~ 2.8 N/m (free resonance frequency ~ 75 kHz) were used.

Molecular dynamics simulation. All simulations for the graphene systems are performed on the largescale atomic/molecular massively parallel simulator (LAMMPS) software⁴³. To obtain the reconstructed tMBG with a linear soliton, we setup the system as three stacked rectangular graphene sheets in a simulation box with non-periodic boundaries. The dangling bonds of edge carbon atoms are saturated by adding hydrogen atoms. The bottom layer is twisted around the sheet center. For the top and middle layers, the AB and BA stacking regions are separated by a 60 Å wide soliton region along an armchair direction. For AB (BA) region, atoms in the top layer are displaced by 1.42 Å (2.84 Å) in the armchair direction relative to the middle layer. The atoms in the top layer are displaced incrementally by 1.42 Å to 2.84 Å as AB changes to BA through the soliton region. In Fig. 2c, the twist angle is 0.5°. The sheets are 1025.44 Å long in the arm chair direction, and 1182.03 Å long in the zigzag direction. The box size is 1252.89 Å $\times$ 1097.97 Å $\times$ 93.3 Å. In Fig. 2f, the twist angle is 0.4°. The sheets are 3326.30 Å long in the armchair direction,

and 3838.83 Å long in the zigzag direction. The box size is 3923.83 Å × 3415.17 Å × 93.3 Å. Figures 2c and 2f has the following crucial difference. The stacking between the top and middle layers in the region left (right) to the linear soliton is BA (AB) in Fig. 2c, but AB(BA) in Fig. 2f. The interactions between C-C and C-H atoms are described by the reactive empirical bond order (REBO) potential⁴⁴. The interlayer interaction between graphene layers is described by the kolmogorov/crespi/z version of Kolmogorov-Crespi interaction potential⁴⁵. We perform energy minimization simulation to obtain the relaxed structure. All atoms were allowed to move freely during the energy minimization. For clear visualization of the relaxed stacking orders, we adopt the stacking identity (StI), defined as:

$$\text{StI}(\mathbf{r}) = \sum_{i=1}^3 \cos[\mathbf{b}_i \cdot \boldsymbol{\delta}_{12}(\mathbf{r})] + \sum_{i=1}^3 \cos[\mathbf{b}_i \cdot \boldsymbol{\delta}_{23}(\mathbf{r})] + \lambda \cdot \sum_{i=1}^3 \cos[\mathbf{b}_i \cdot \boldsymbol{\delta}_{13}(\mathbf{r})]$$

where $\mathbf{r}$ is the position vector in the x - y plane; $\mathbf{b}_1 = 2\pi(1, 1/\sqrt{3})/a$, $\mathbf{b}_2 = 2\pi(-1, 1/\sqrt{3})/a$, $\mathbf{b}_3 = -\mathbf{b}_1 - \mathbf{b}_2$ are the reciprocal lattice vectors of graphene, $a = 2.46$ Å is the lattice constant of graphene; $\boldsymbol{\delta}_{ij}(\mathbf{r}) = \boldsymbol{\delta}_{0,ij}(\mathbf{r}) + \mathbf{u}^{(j)}(\mathbf{r}) - \mathbf{u}^{(i)}(\mathbf{r})$ are relative in-plane shift vector between the i -th and j -th layer, $\boldsymbol{\delta}_{0,ij}(\mathbf{r})$ are the shift vector before relaxation, $\mathbf{u}^{(i)}(\mathbf{r})$ are relaxation displacement fields of the i -th layer; λ is a weight coefficient to quantify the stacking identity between first and third layers.

Stacking energy analysis. The stacking energy of Bernal stacking (E_b for AB-A or BA-B) is lower than that of rhombohedra stacking (E_r for AB-C and BA-C). In Fig. 2b, the relation between total stacking energy of tMBG (E) and lateral position fraction of the linear solitons ($0 \leq x \leq 1$) can be formulated as: $E \propto \alpha x^2 + \beta x + \gamma$, where $\alpha = 2(E_r - E_b)/\sqrt{3}$, $\beta = 2(E_b - E_r)/\sqrt{3}$, and $\gamma = (E_b + E_r)/\sqrt{3}$. In Fig. 2e, the coefficients are $\alpha = 2(E_b - E_r)/\sqrt{3}$, $\beta = 2(E_r - E_b)/\sqrt{3}$, and $\gamma = (E_b + E_r)/\sqrt{3}$, respectively.

Density functional theory calculations. DFT calculations were performed using the Vienna Ab initio Simulation Package (VASP) code, with the Perdew–Burke–Ernzerhof parametrization (PBE) of the generalized gradient approximation (GGA) and projector augmented wave (PAW) potentials. The multilayer graphene structures of different stacking forms were optimized using triclinic

(rhombic) simulations cells with a cell dimension of 2.46 Å. The plane-wave kinetic energy cutoff was set to 500 eV, and a vacuum region larger than 15 Å was adopted. Structures were fully relaxed until the force on each atom was less than 0.01 eV/Å. The x-y coordinates of the carbon atoms were fixed and the z coordinate was allowed to relax. All results were calculated with the van der Waals corrections applied with the DFT-D2 method of Grimme⁴⁶.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at <http://pubs.acs.org>.

Samples fabrication processes, removing background signals in LPFM, and determining the in-plane deformation of a linear soliton in tMBG.

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Z.F., Yi Shi and X.W. supervised the project. Y.L. and H.Z. fabricated the devices. Y.L. and Y.W. performed the AFM measurements. H.M. and F.W. performed the MD simulations. M.X. and Z.Z. performed the DFT calculations. K.W. and T.T. provided the bulk BN crystals. Z.F., Y.L., H.M., and F.W. analyzed the data. Z.F. and Y.L. wrote the paper with input from all authors.

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Notes

The authors declare no competing financial interest.

ACKNOWLEDGMENTS

This work is supported by the National Key Research and Development Program of China (2021YFA0715600), National Natural Science Foundation of China (12274222 and 12404217), the National Science Foundation of Jiangsu Province (BK20220756) and the Fundamental Research Funds for the Central Universities (2024300420). F.W. is supported by National Natural Science Foundation of China (12274333) and National Key Research and Development Program of China (2021YFA1401300 and 2022YFA1402401). The MD simulations in this paper have been performed on the supercomputing system in the Supercomputing Center of Wuhan University. M.X. and Z.Z. are supported by National Natural Science Foundation of China (12102180 and 12261160367). Yan Shi is supported by National Natural Science Foundation of China (12072150) and National Natural Science Foundation of China for Creative Research Groups (51921003). K.W. and T.T. acknowledge support from the JSPS KAKENHI (21H05233 and 23H02052) and World Premier International Research Center Initiative (WPI), MEXT, Japan.

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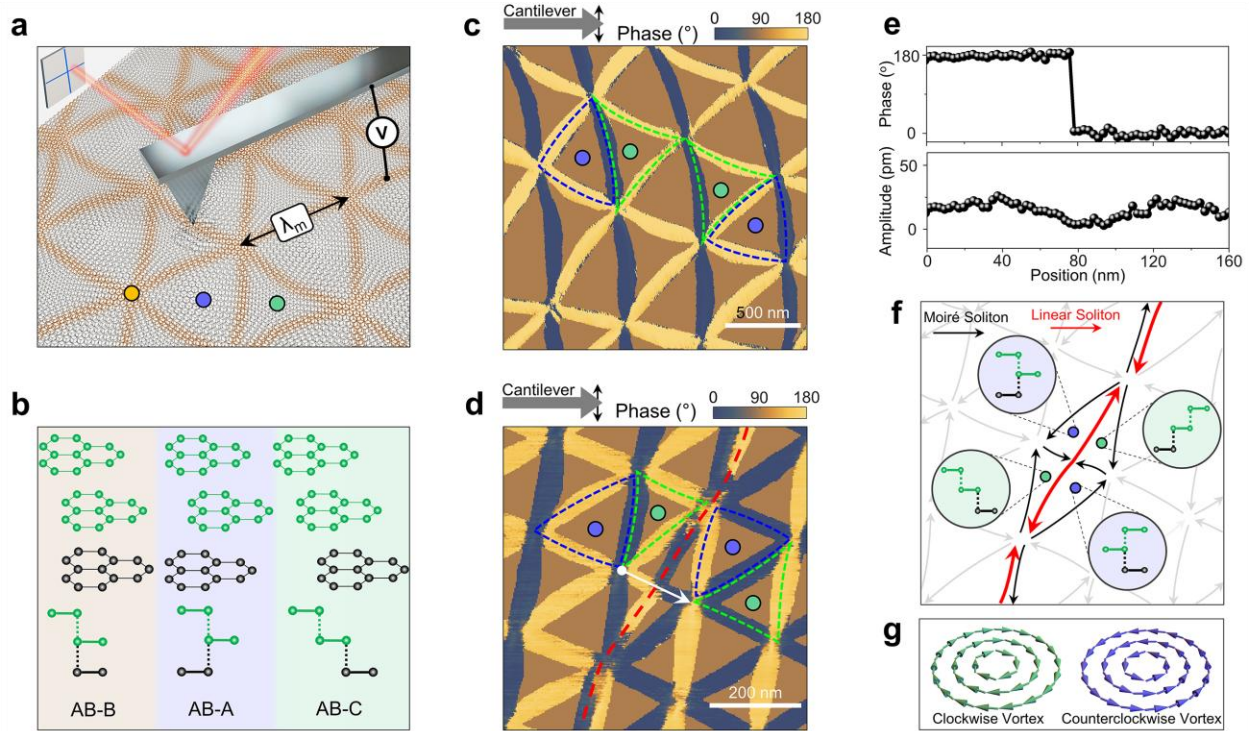


Figure 1. Coexistence of linear and moiré solitons in tMBG. **a**, Schematic setup for LFPF measurement on a marginally tMBG moiré superlattice. **b**, Stacking configurations of AB-B, AB-A and AB-C stacking domains. **c**, Decoupled LFPF phase image of a typical tMBG (tMBG1), exhibiting strong contrasts near moiré solitons. The convex blue and concave green dashed triangles outline the Bernal and rhombohedral stacking domains, respectively. **d**, Decoupled LFPF phase image of tMBG2, exhibiting strong contrasts near the red dashed line (a linear soliton) in addition to moiré solitons. **e**, Linecuts of LFPF phase and amplitude along the white arrowed line in **d**. **f**, Stacking configurations of the domains near the red dashed line in **d**. The black and red arrows in **f** denote the in-plane deformation or polarization of the moiré and linear solitons, respectively. The yellow, blue and green dots in **a,c,d,f** denote AB-B/BA-A, Bernal (AB-A/BA-B) and rhombohedral (AB-C/BA-C) stacking domains, respectively. **g**, Schematics of clockwise and counterclockwise vortices.

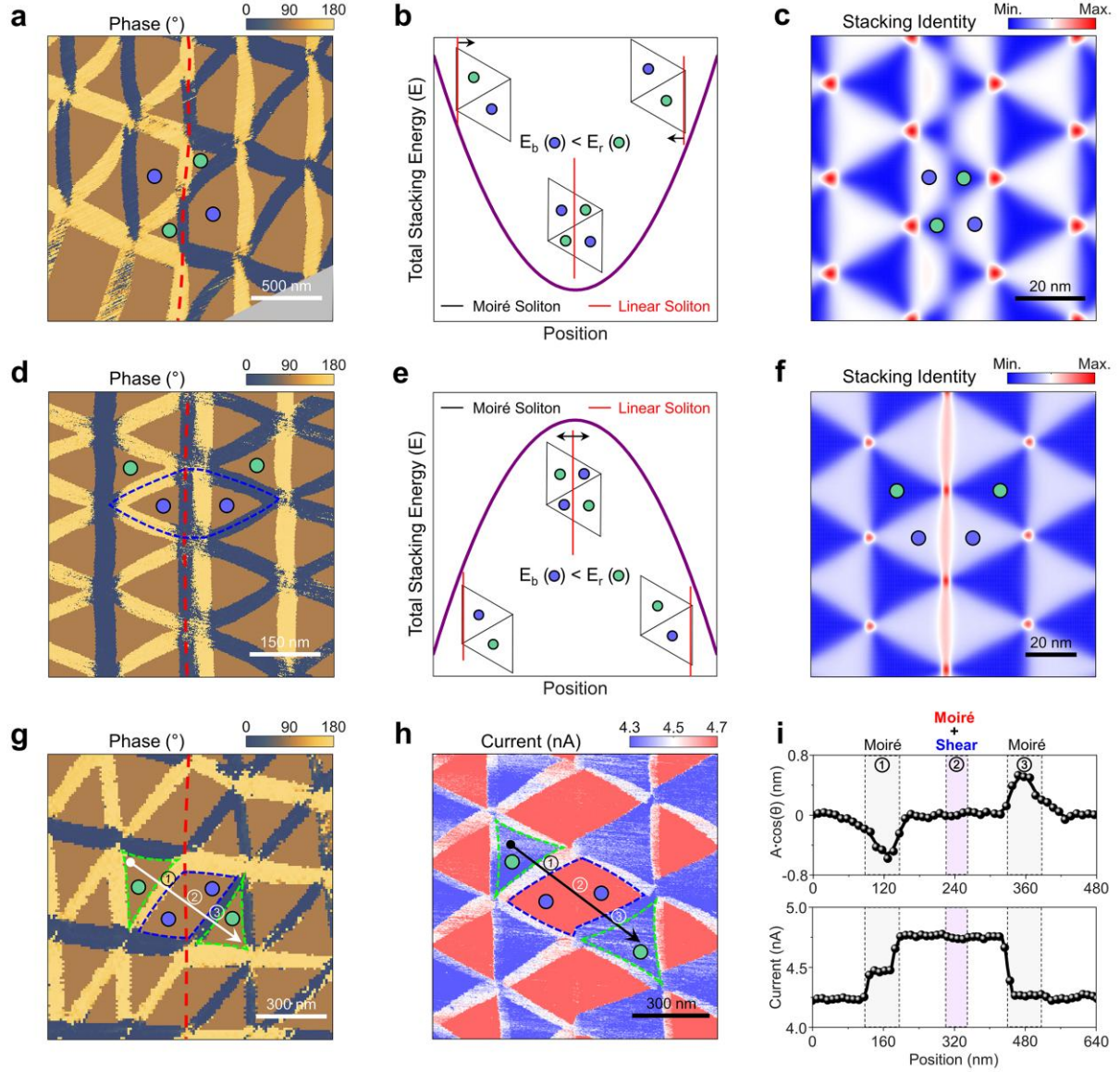


Figure 2. Interaction of linear and moiré solitons in tMBG. **a,d,** Decoupled LPFM phase images of tMBG samples (tMBG3 and tMBG4) with parallel linear and moiré solitons from different interfaces. The red dashed lines denote the linear solitons. **b,e,** Total stacking energies of tMBG as a function of lateral position of the linear solitons with the stacking configurations in **a** and **d**, respectively. The stacking energy of Bernal stacking (E_b for AB-A or BA-B) is lower than that of rhombohedra stacking (E_r for AB-C and BA-C). **c,f,** Corresponding molecular dynamics simulations in **a** and **d**. **g,** Decoupled LPFM phase image of tMBG5 with parallel linear and moiré solitons from the same (twisted) interface. The red dashed line denotes the linear soliton. **h,** In-situ cAFM current image of the same sample in **g** with slight distortions. **i,** Linecuts of the in-phase LPFM signals ($A \cdot \cos(\theta)$) and cAFM current along the white and black arrowed lines in **g** and **h**, respectively, where A and θ are decoupled amplitude and phase of LPFM signal. The blue and green dots in **a-h** denote Bernal and rhombohedral stacking domains, respectively.

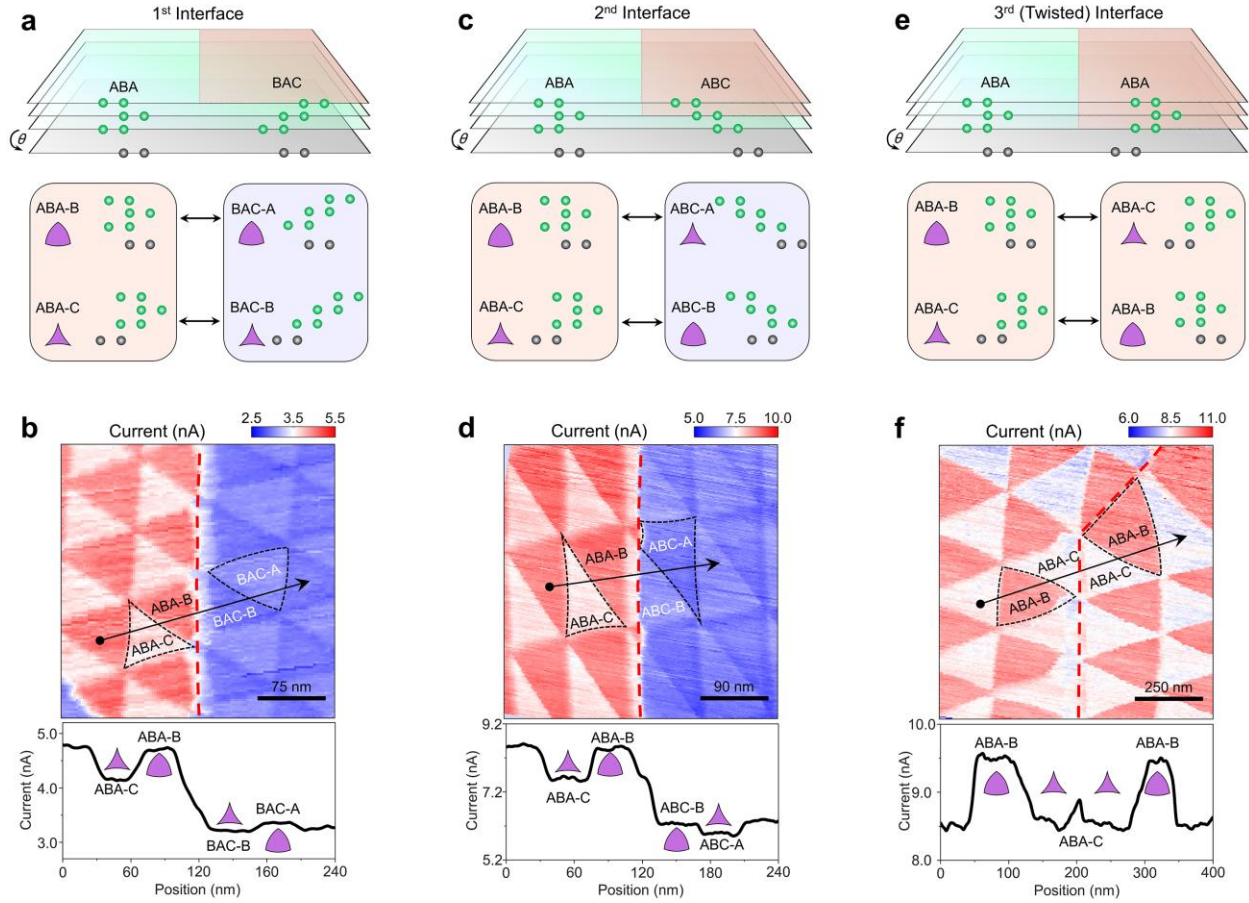


Figure 3. Interface-resolved linear solitons in tMTG. **a,c,e**, Schematics of tMTG with linear solitons located at the first, second and third (twisted) interfaces and corresponding changes of stacking orders (lower insets). The curvatures of different stacking domains (concave and convex triangles) are inferred from their relative stacking energies. **b,d,f**, Current images and corresponding linecuts of three tMTG samples (tMTG1, tMTG2 and tMTG3), corresponding to linear solitons from different interfaces, as illustrated in **a,c,e** respectively. The lower insets are linecuts along the black arrowed lines in the main figures. The black dashed triangles serve as guidelines for identifying convex or concave domains across linear solitons.

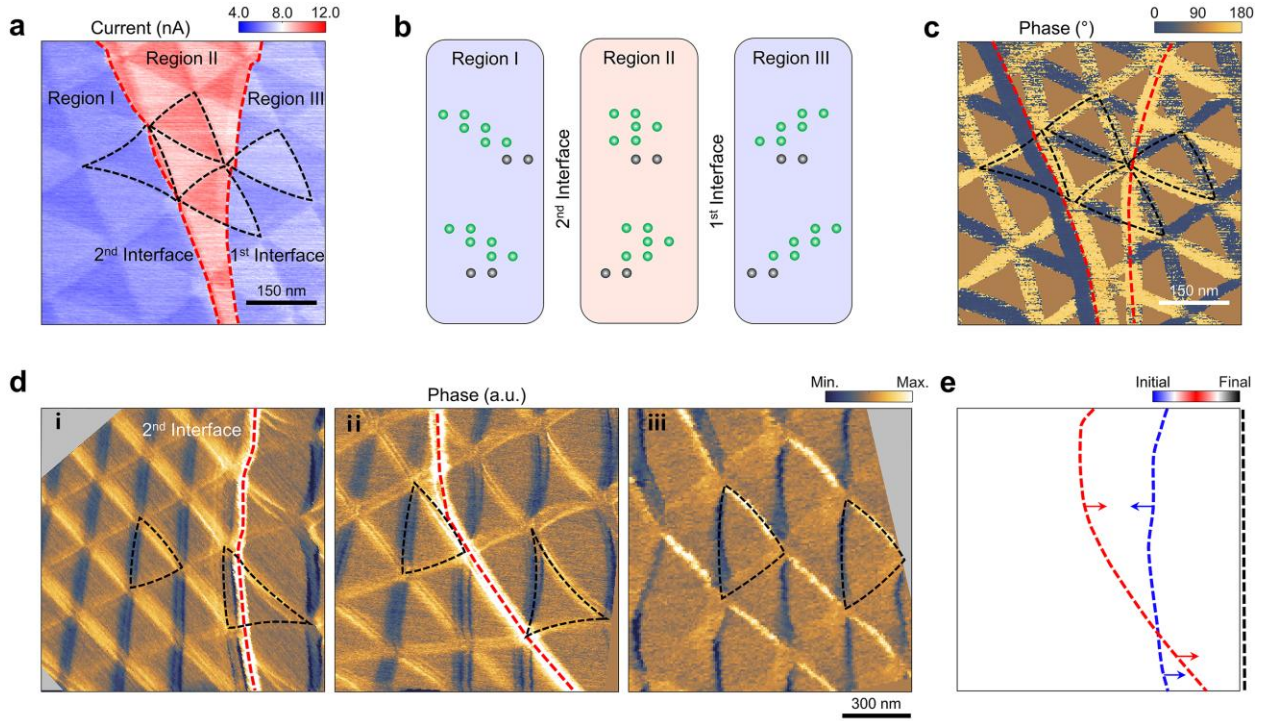


Figure 4. LPFM images and motions of linear solitons in tMTG. **a**, Current image of tMTG4, exhibiting two linear solitons located at the first and second interfaces simultaneously. **b,c**, Stacking configurations and in-situ LPFM phase of the corresponding regions in **a**. **d**, Selected frames of LPFM phase of tMTG5, exhibiting a linear soliton at the trilayer interface. **e**, Schematic positions of the linear soliton at different stages. The arrows mark the directions of the subsequent motions. The black dashed triangles in **a,c,d** serve as are guidelines for identifying the same convex or concave domains.